Signal Conditioning Basics using Op-amps

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In most real-world control applications, we need to use sensors to transform physical variables (heat, pressure, motor shaft position, etc.) to electrical variables (voltages or currents). Only then the signals could be sampled and processed by a computer. As it normally happens, the signal output from a sensor might not be clean enough to accurately represent the quantity it measures. Very likely that it would be contaminated by high-frequency noise, or undesired disturbance with fixed frequency (interference from 50 Hz household appliance, for example). At the other analog-digital junction, the direct signal from DAC module might need to be smoothen by a low-pass filter. For a low-cost application, a passive circuit may be used for signal conditioning purpose. The drawback is lack of impedance buffering and signal amplification. If your sensor or DAC has a limited sourcing capability, an active circuit using operational amplifier (op-amps) is a better choice.

This brief technical article summarizes the use of op-amps in certain signal conditioning circuits. We only provide basic derivations, circuit examples, and simulation results, leaving detailed analysis to standard textbooks.

Open-loop op-amp model

Figure 1 shows a symbol and open-loop model of an op-amp. The device has 2 input ports, named inverting (-) input and non-inverting (+) input. The output is simply an amplified signal of the difference between the two inputs

$$v_{out} = A_{V(OL)}\left(v^{+} - v^{-}\right) \qquad (1)$$

Where $A_{V(OL)}$ is called *open-loop gain*. In a typical op-amp this gain could be as high as 10^5 to 10^7. This is way too high to be used as a linear amplifier. Later on we will show how feedback is used to adjust the op-amp gain to a desired value. Figure 1 also shows the input and output impedances of the op-amp. The input impedance is normally very high while the output impedance is very low. The actual values vary with products.

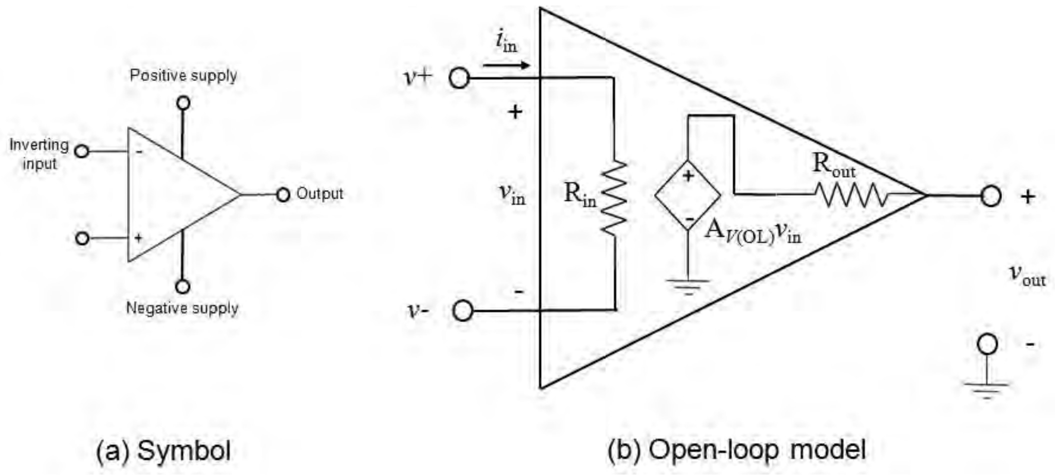

Figure 1 Symbol and open-loop model of an op-amp

Inverting amplifier

Perhaps the easiest way to build a linear amplifier from an op-amp is an *inverting amplifier* shown in Figure 2, where an input signal is connected to the inverting input pin via a resistor R_S. The non-inverting input is connected to ground. There is also a feedback resistor R_F from output to the inverting input. We now proceed to show that the gain of this circuit could be set by choosing the resistors R_S and R_F. Consider the node at the inverting input. From Kirchoff's Current Law (KCL), we have

$$i_S + i_F = i_{in} \qquad (2)$$

Using the fact that the input impedance of the op-amp is very large, the current flowing into the device is negligible. By Ohm's Law, all the currents in (2) are described as

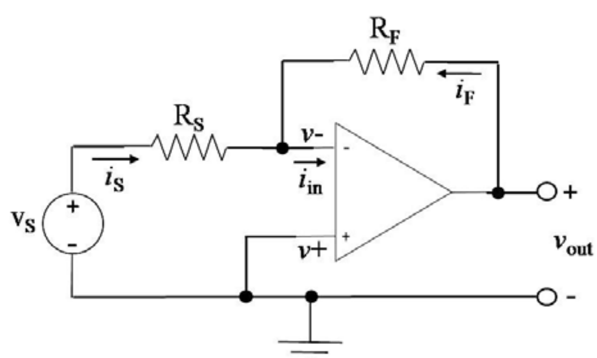

Figure 2 an inverting amplifier

$$i_{in} = 0 \quad i_S = \frac{v_S - v^-}{R_S} \quad i_F = \frac{v_{out} - v^-}{R_F} \tag{3}$$

Next, consider the open-loop model (with the non-inverting input connected to ground)

$$v_{out} = A_{V(OL)}\left(v^+ - v^-\right) = -A_{V(OL)}v^- \tag{4}$$

Therefore,

$$v^- = -\frac{v_{out}}{A_{V(OL)}} \tag{5}$$

What we want to find is the ratio v_{out}/v_S. This quantity is called *closed-loop gain*, because of the feedback from output to input. From (2) and (3) we get

$$i_S = -i_F \tag{6}$$

Substitute terms from (3) and rearrange

$$\frac{v_S}{R_S} + \frac{v_{out}}{A_{V(OL)}R_S} = -\frac{v_{out}}{R_F} - \frac{v_{out}}{A_{V(OL)}R_F}$$

$$\frac{v_S}{R_S} = -\frac{v_{out}}{R_F} - \frac{v_{out}}{A_{V(OL)}R_F} - \frac{v_{out}}{A_{V(OL)}R_S}$$

$$v_S = -v_{out}\left(\frac{1}{R_F/R_S} + \frac{1}{A_{V(OL)}R_F/R_S} + \frac{1}{A_{V(OL)}}\right) \tag{7}$$

This equation can be simplified further by noting that the open-loop gain $A_{V(OL)}$ of op-amp is very high, so the second and third terms in parenthesis on the right side of (7) is insignificant compared to the first term. Hence the closed-loop gain of inverting amplifier can be approximated by

$$\boxed{\frac{v_{out}}{v_S} = -\frac{R_F}{R_S}} \tag{8}$$

Observe that the closed-loop gain is determined by the values of two resistors. So an amplifier with arbitrary gain can be conveniently designed. Also notice the minus sign in (8). This indicates that the input and output are 180^o out of phase, hence the name inverting amp. Another drawback is the input impedance depends on the values of R_S and R_F. For real applications the values should not be chosen too low. Too high values, on the other hand, could make your amp noisy. Resistors in the range of 1KΩ – 1 MΩ are practical choices.

We leave this section with some further observation that will be used later. From (5), with the fact that the open-loop gain $A_{V(OL)}$ is very high, the voltage at inverting input must be infinitesimal. So it is rational to make an assumption that for the inverting amp, the voltage at inverting input is approximately zero

$$v^- \approx 0 \tag{9}$$

Moreover, it is left as an exercise to show that, in an op-amp circuit with negative feedback from output to inverting input, the result of feedback is to force the voltages at both inputs of op-amp to become equal

$$v^- \approx v^+ \tag{10}$$

Ex. 1: An op-amp model is conveniently provided in Modelica software. Figure 3 shows how to construct a Scicos/Modelica diagram for circuit simulation. Choosing R_S = 1 KΩ and R_F = 4 KΩ yields a gain of -4. The input is a sinusoid with unit magnitude and 1 Hz frequency. Figure 4 shows the input and output waveforms captured from the scope. The output has amplification of 4 and opposite phase to the input. This conforms to the derivation given above.

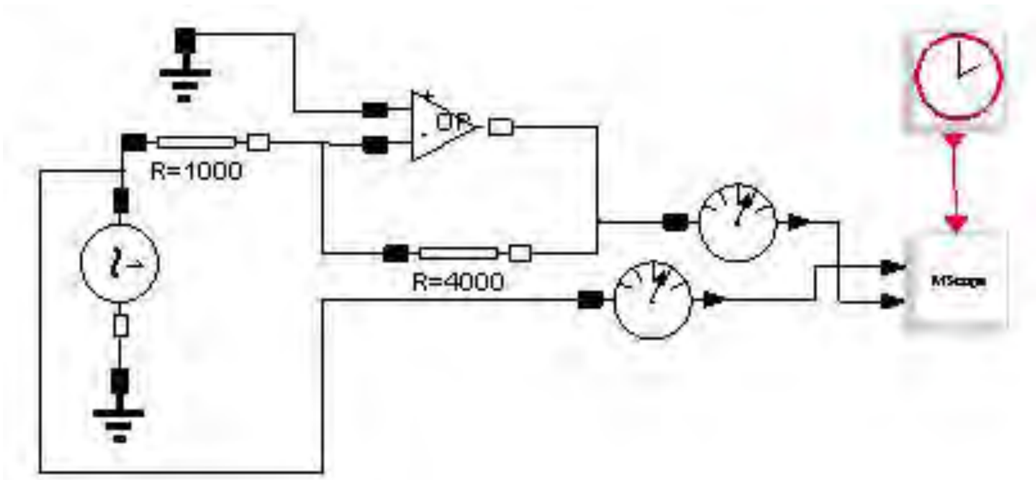

Figure 3: Simulation of an inverting amp using Scicos/Modelica

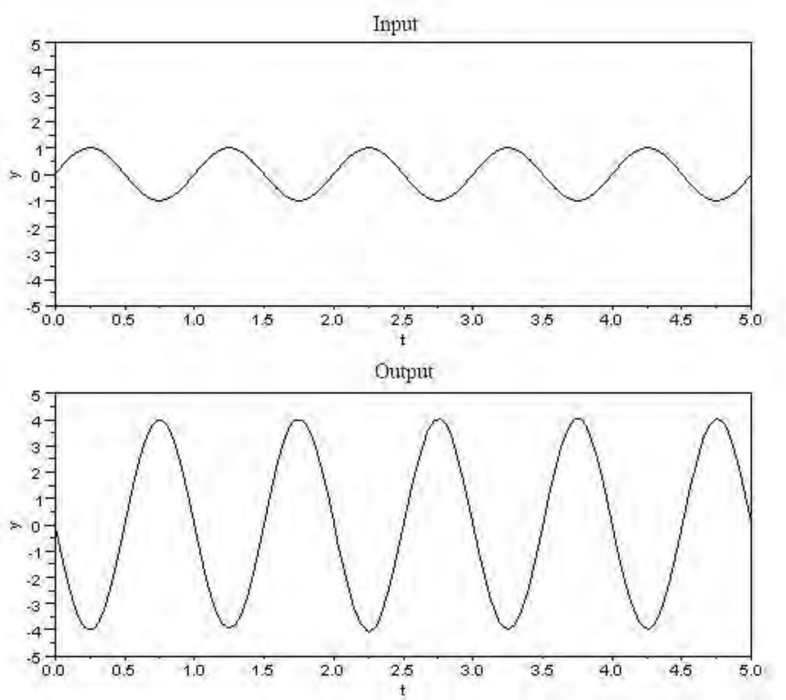

Figure 4: input and output waveforms from simulation

Summing amplifier

The inverting amplifier discussed so forth could be augmented to another useful circuit called a *summing amplifier*, or *mixer*, shown in Figure 5, when more than one signal sources are connected to the inverting input. Applying KCL at the inverting input node, we have

$$i_1 + i_2 + \ldots + i_N = -i_F \tag{11}$$

where the current from each source can be computed as

$$i_n = \frac{v_{Sn}}{R_{Sn}} \qquad n = 1,2,\ldots,N \tag{12}$$

and the feedback current from output equals

$$i_F = \frac{v_{out}}{R_F} \tag{13}$$

Combining (12), (13) and using (6) yields

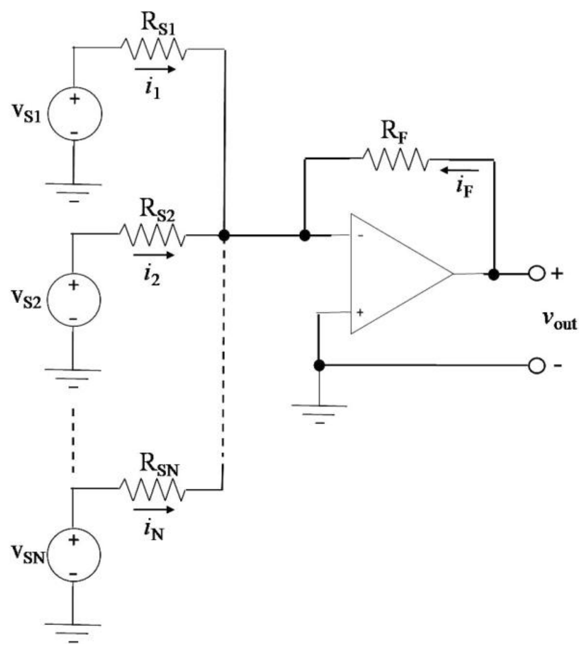

Figure 5 a summing amplifer

$$\sum_{n=1}^{N} \frac{v_{Sn}}{R_{Sn}} = -\frac{v_{out}}{R_F} \qquad (14)$$

or

$$\boxed{v_{out} = -\sum_{n=1}^{N} \frac{R_F}{R_{Sn}} v_{Sn}} \qquad (15)$$

In words, the output equals weighed summation of all inputs. The weight for each input is determined from the ratio of feedback resistor to the resistance at that source.

Non-inverting amplifier

In applications where phase inversion is undesirable, we can use a *non-inverting amplifier* shown in Figure 6. Note that the input signal is fed to the non-inverting input pin. The circuit can be analyzed using quite the same approach as before. Using KCL at inverting input node yields

$$i_F = i_S + i_{in} \approx i_S \qquad (16)$$

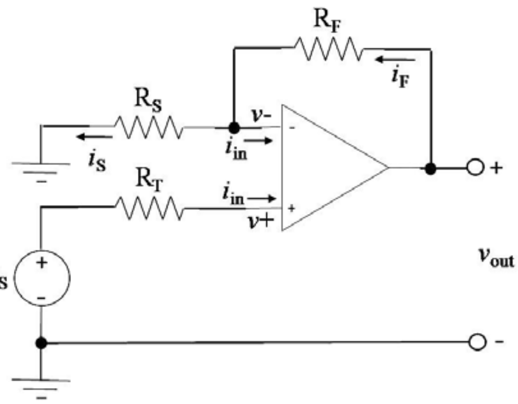

Figure 6 a non-inverting amplifier

where

$$i_F = \frac{v_{out} - v^-}{R_F} \tag{17}$$

$$i_S = \frac{v^-}{R_S} \tag{18}$$

and since $i_{in} = 0$ causes the voltage across R_T equals zero. Hence,

$$v^+ = v_S \tag{19}$$

Using (10),

$$v^- = v^+ = v_S \tag{20}$$

After substituting (20) into (16), (17), it is straightforward to show that

$$i_F = i_S \tag{21}$$

or

$$\frac{v_{out} - v_S}{R_F} = \frac{v_S}{R_S} \tag{22}$$

After rearrangement, we get the closed-loop gain of non-inverting amplifier as

$$\boxed{\frac{v_{out}}{v_S} = 1 + \frac{R_F}{R_S}} \tag{23}$$

Notice that the gain of non-inverting amplifier is positive, and has minimum value equal 1. This means the output is in phase with the input, and the circuit cannot be used for signal attenuation.

Remark: The derivation of (23) above uses the assumption (10), $v^- \approx v^+$, to simplify the process. A more straightforward way like the case of inverting amplifier is also possible.

Ex. 2: A non-inverting amplifier can be constructed in Scicos/Modelica as shown in Figure 7. To compare with Ex.1, we choose R_S = 1 KΩ and R_F = 3 KΩ . From (23), the resulting gain equals 4. The value of R_T has no effect on the gain. In this example we simply choose R_T = R_S. The simulation result is shown in Figure 8. The output from the scope indicates amplification of 4, conforming to the computation from (23). What is different from the inverting amp case in Ex. 1 is now the input and output waveform are in phase.

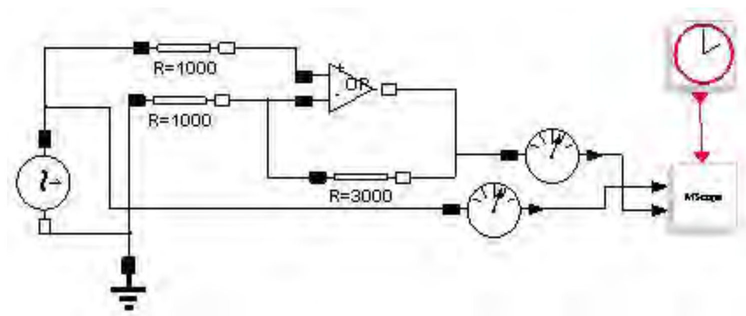

Figure 7: Simulation of a non-inverting amp using Scicos/Modelica

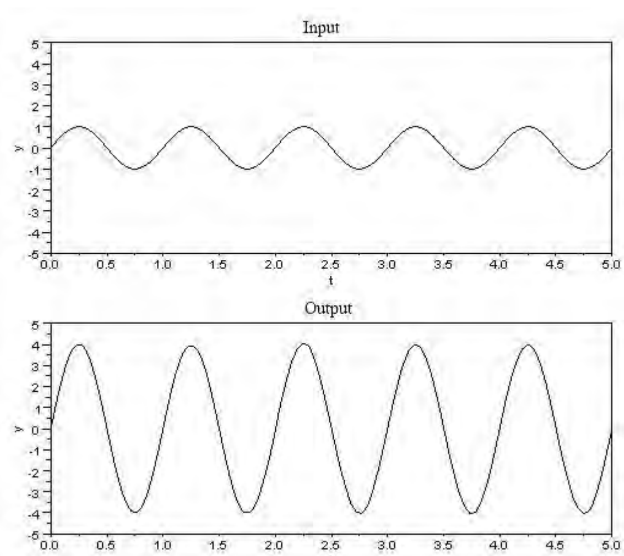

Figure 8: Input and output waveforms from simulation

Voltage follower

A variation of non-inverting amp is known as a *voltage folllower*. The gain of this circuit equals 1. The large input impedance and small output impedance of the op-amp makes it suitable as a buffer between a source with high output impedance and a load. As shown in Figure 9, the voltage follower does not need any external resistance to operate. The gain computation still conforms to (23), of course, with $R_F = 0$ (short circuit) and $R_S = \infty$ (open circuit).

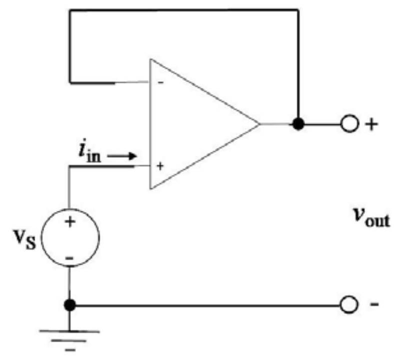

Figure 9: a voltage follower circuit

Differential amplifier

The inverting and non-inverting properties of an op-amp can be combined together to a *differential amplifer* shown in Figure 10. This circuit is used to amplify the difference of two input signals. The two inputs are connected to two independent sources, most often with opposite phases. The four resistors are selected in pairs, so are labeled only as R_1 and R_2.

An analysis of this circuit can be conveniently done using the superposition principle, which states that the output is a combination of the results when each voltage source acting alone. We

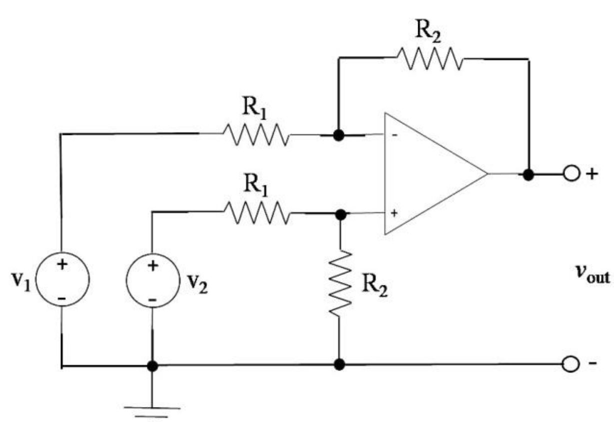

Figure 10: a differential amplifier

start by finding the output from v_1 alone (replacing v_2 with a short circuit wire). This output equals

$$v_{out1} = -\frac{R_2}{R_1} v_1 \tag{24}$$

Next we find the output from v_2 alone. Using (23) together with voltage divider, this output equals

$$v_{out2} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_2}{R_1 + R_2}\right) v_2 = \frac{R_2}{R_1} v_2 \tag{25}$$

By the superposition principle, the whole output equals $v_{out} = v_{out1} + v_{out2}$. Combining (24) and (25) yields

$$\boxed{v_{out} = \frac{R_2}{R_1}(v_2 - v_1)} \tag{26}$$

A differential amp is commonly used as a noise rejection circuit. Suppose we want to transmit a signal through a long cable. The signal could easily contaminated by noise and interference. The scheme is to send two signals at the transmitter, called the *differential mode*, where the second signal is the complement (or 180 degree out of phase) with the original signal. If the two wires are kept close together (e.g., in the same cable), any interference noise mixed into the signals is likely to have roughly the same amplitude and phase, called the *common mode*, and thus can be eliminated by a differential amp at the receiver end. The performance of an op-amp in rejecting the unwanted common mode can be measured by its *common mode rejection ratio (CMRR)*, which is indicated in the datasheet of a standard op-amp.

Ex.3: When an electronic device is subjected to a random interference signal, it is problematic to design a filter that could get rid of such interference since it has a very broad frequency spectrum. So a better approach is to use a differential amp. Figure 11 shows a simulation of differential amp in a ECG (Electrocardiography device) using Scicos/Modelica. The desired 2 Hz signal from ECG has unit amplitude. The noise signal is constructed using random generator block. Values of resistors R_1 and R_2 in (26) are chosen as 5KΩ and 10KΩ, respectively. Figure 12 shows the contaminated signals entering the two inputs of op-amp. Figure 13 shows the output signal with the common mode eliminated. One can easily verify that the gain conforms to (26) with the chosen values of R_1 and R_2 .

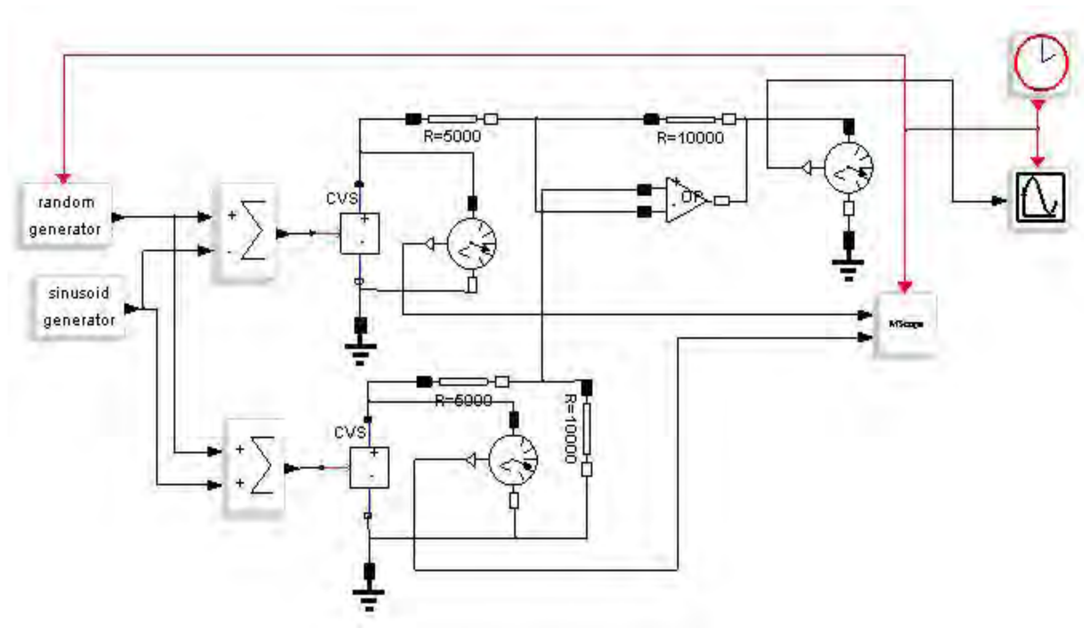

Figure 11: Scicos/Modelica diagram of a differential amp used in ECG device

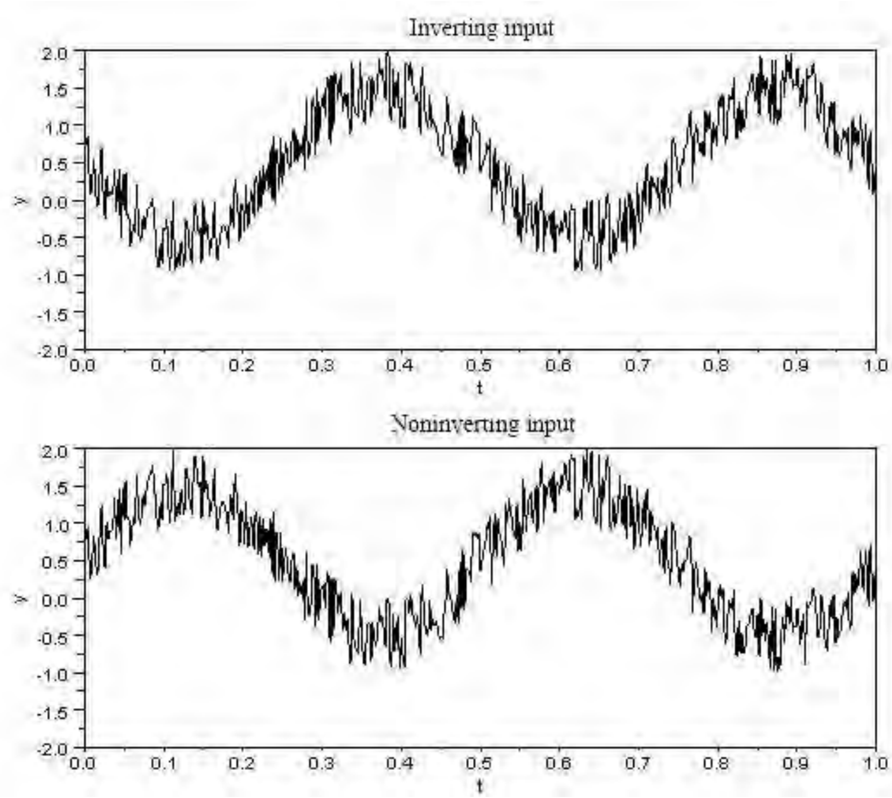

Figure 12: contaminated signals at the two inputs of op-amp

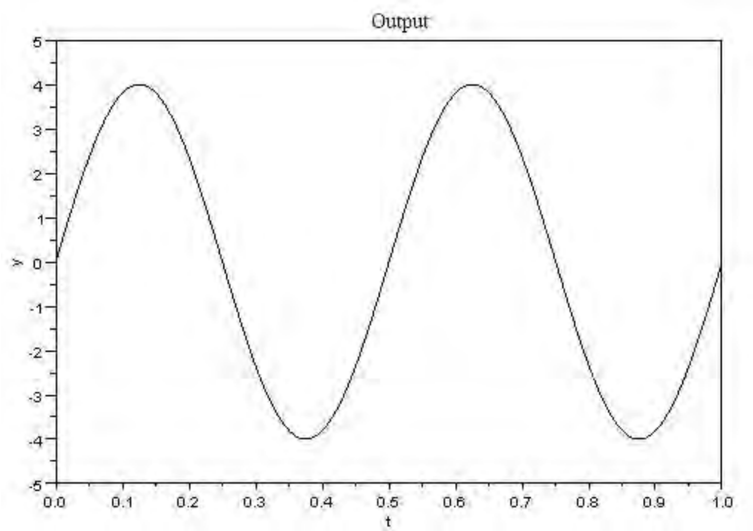

Figure 13: output signal from the differential amp

Level shifter

In certain applications, we have to deal with signals that are not purely AC, but may have some DC components as well. Or it could happen that we want the output waveform of op-amp to have non-zero mean; i.e., to swing on a DC offset. An op-amp circuit used in these circumstances is called a *level-shifter*. The design of this circuit does not have a fixed formula but has to be tailored to the problem at hand. Nevertheless, it relies on the same principle of the differential amp discussed earlier. We demonstrate this with an example.

Ex. 4: Suppose we want to design a servomotor drive that accepts analog input in the range $\pm$ 10 volts. A 10 V command makes the motor rotate clockwise at maximum speed, say, 3,000 rpm, while a -10 V drives the motor counterclockwise at maximum speed as well. At 0 V command the motor stops spinning. This analog command input must be sampled by the ADC module of a microcontroller that has analog input range 0 – 5 Volts. Obviously, we cannot connect the analog command directly to ADC pin of the microcontroller. So a level shifter circuit like shown in Figure 13 is designed to attenuate and shift the $\pm$ 10 V command signal to 0 – 5 V range. Our job is to select the values for $R_1 - R_4$, and V_B appropriately.

Once again, the superposition principle is used in the design. Let v_{in} = 0 for the moment. We want to create the 2.5 V DC bias that the AC command signal will ride on. This can be done by selecting $R_1 = R_2$ and V_B = -2.5 V. Now, we want to make the AC output part of the op-amp swing within $\pm$ 2.5 V range. Observe that with $V_B = 0$, $R_1 = R_2$ yields the non-inverting gain of

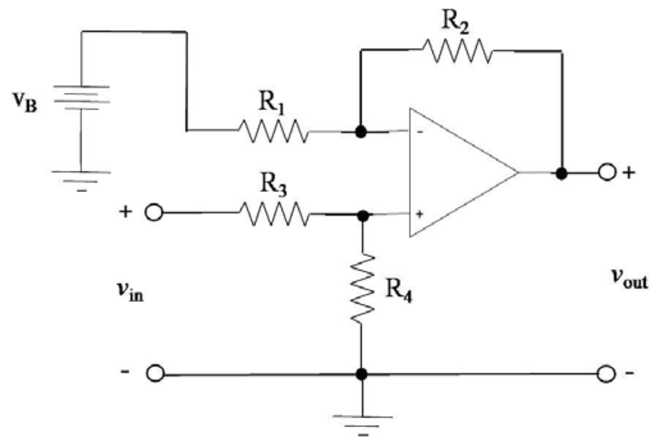

Figure 14: A level-shifter circuit

2. So the signal at non-inverting pin must equal ± 1.25 V. Using the voltage divider formula, we have

$$1.25 = \frac{R_4}{R_3 + R_4}(10) \tag{27}$$

or

$$R_3 = 7R_4 \tag{28}$$

To verify our design, we simulate this level-shifter in Scicos/Modelica. Simply select $R_1 = R_2 = R_4 = 10$ KΩ and $R_3 = 70$ KΩ, $V_B = -2.5$ V. Figure 15 shows the block diagram and Figure 16 shows the resulting input and output waveforms.

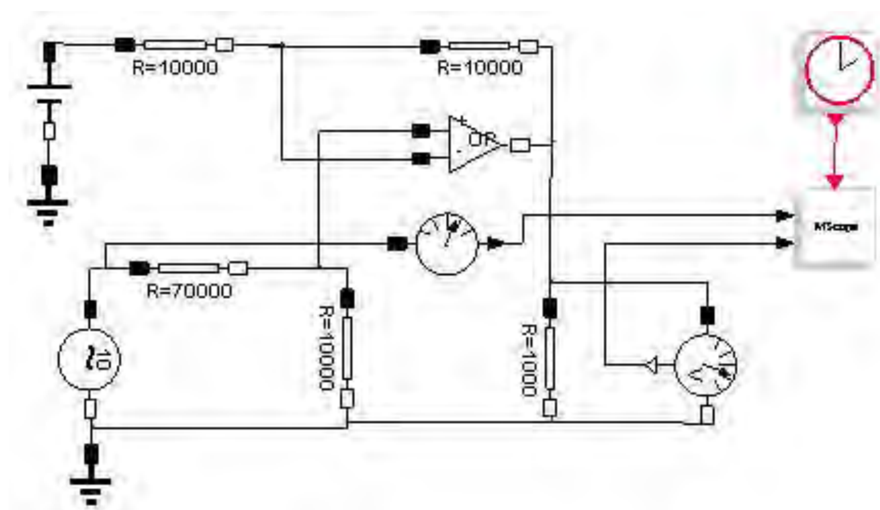

Figure 15: Simulation of a level-shifter using Scicos/Modelica

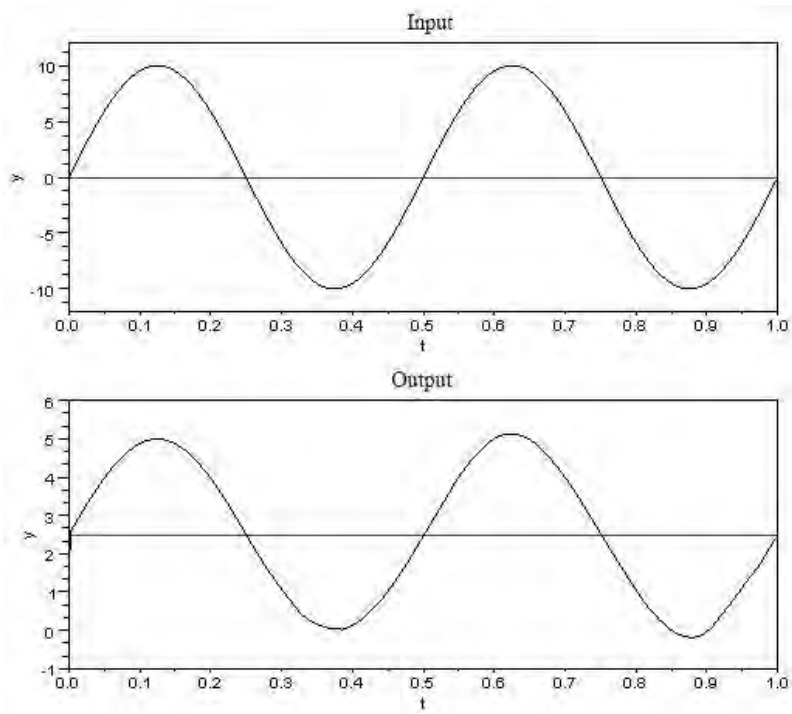

Figure 16: input and output waveforms from the level-shifter

Active Filters

A filter is an electronic circuit that allows only selected frequency region to pass through. Simplest filters can be constructed from passive components alone. When impedance matching becomes a problem, or amplification is needed, a circuit designer would switch to an active filter counterpart. In this section we discuss how to construct some common active filters using op-amps.

To understand how active filters work, we first mention that the principles of inverting and non-inverting amps remain the same, with resistors replaced by a more general impedance blocks like shown in Figure 17. The gain equations are still in the same form, using impedance in place of resistance. For the inverting amp case, we have

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_S}(j\omega) = -\frac{Z_F}{Z_S} \tag{29}$$

and for the non-inverting amp case,

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_S}(j\omega) = 1 + \frac{Z_F}{Z_S} \tag{30}$$

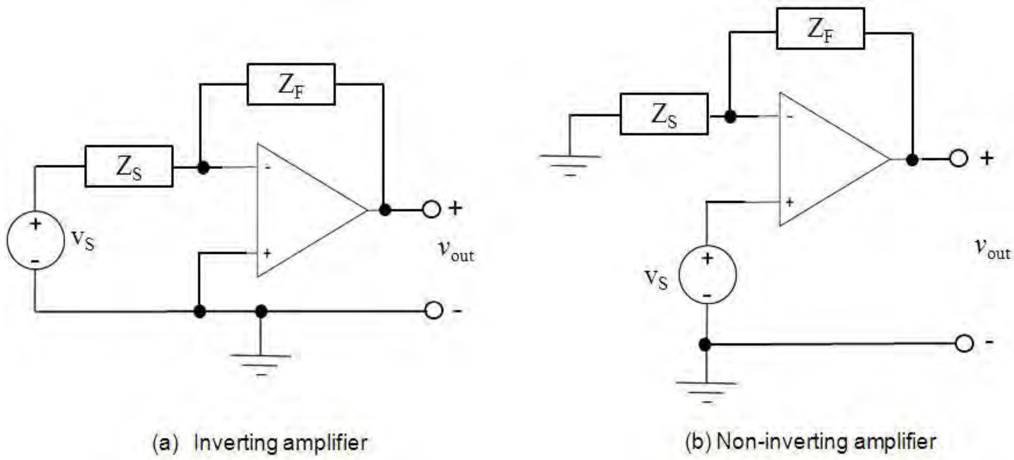

Figure 17: op-amp circuits with impedance

Note that now the gains are complex numbers that vary with frequency. Therefore, frequency-dependent gain of a filter can be constructed by selecting Z_S and Z_F. Their values can be dictated by basic passive components; i.e., resistors, capacitors, and inductors.

Active Low-Pass Filter

Figure 18 shows how to construct an *Active Low-Pass Filter (ALPF)* using an op-amp and a capacitor in parallel with a resistor on its negative feedback path. The gain of this circuit can be computed using (29), where

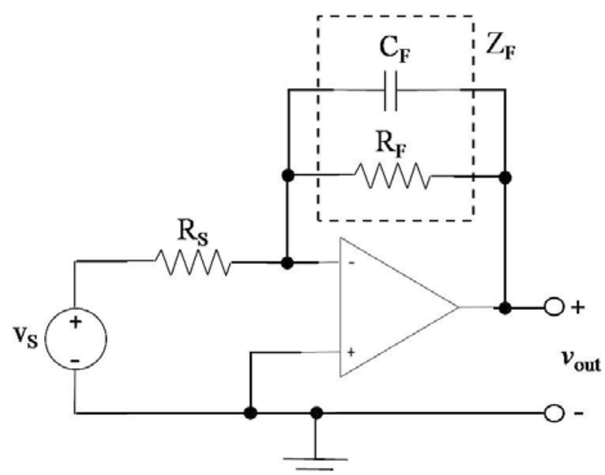

Figure 18: Active Low-Pass Filter (ALPF)

$$Z_S = R_S \tag{31}$$

$$Z_F = R_F \parallel \frac{1}{j\omega C_F} = \frac{R_F}{1 + j\omega C_F R_F} \tag{32}$$

So the gain of ALPF can be computed as

$$\boxed{A_{LPF}(j\omega) = -\frac{Z_F}{Z_S} = -\frac{R_F / R_S}{1 + j\omega C_F R_F}} \tag{33}$$

with cutoff frequency

$$\boxed{\omega_o = \frac{1}{R_F C_F}} \tag{34}$$

Figure 19 shows frequency response of ALPF with $R_S = 1$ KΩ, $R_F = 10$ KΩ, and $C_F = 100$ μF. These values give amplification of 10 in low frequency region. The cutoff frequency equals 1 rad/s. We plot both the absolute gain (above), and gain in decibel (below), which is computed using the formula

$$|A_{LPF}(j\omega)| = 20 \log_{10} A_{LPF}(j\omega) \tag{35}$$

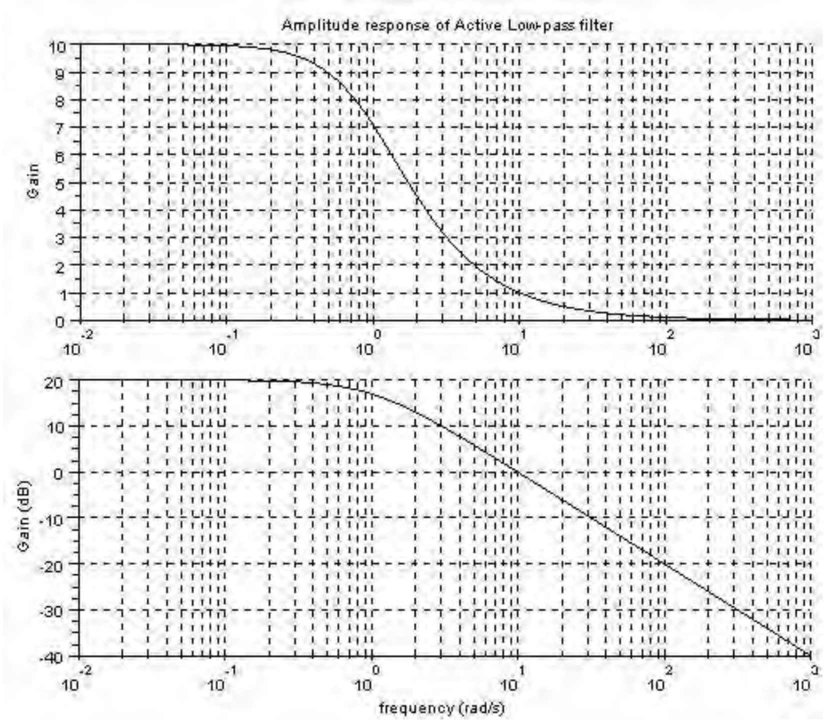

Figure 19: frequency response of ALPF

The gain (dB) curve clearly demonstrates that the gain at cutoff frequency decreases by 3 dB, and at frequency above cutoff, the gain decreases with slope – 20 dB/decade.

Active High-Pass Filter

An *Active High-Pass Filter (AHPF)* can be constructed as shown in Figure 20, where the impedances can be computed as

$$Z_S = R_S + \frac{1}{j\omega C_S} \tag{36}$$

$$Z_F = R_F \tag{37}$$

Hence, the gain of this AHPF can be computed as

$$\boxed{A_{HPF}(j\omega) = -\frac{Z_F}{Z_S} = -\frac{j\omega C_S R_F}{1 + j\omega R_S C_S}} \tag{38}$$

when $\omega \to 0$ we have $A_{HPF}(j\omega) \to 0$. On the other hand, when $\omega \to \infty$,

$$\lim_{\omega \to \infty} A_{HPF}(j\omega) = -\frac{R_F}{R_S} \tag{39}$$

This verifies the circuit as a high-pass filter. The frequency response when choosing R_S = 10 KΩ, C_S = 100 μF and R_F = 100 KW is shown in Figure 21. The high frequency gain equals 10 and cutoff frequency equals 1 rad/s.

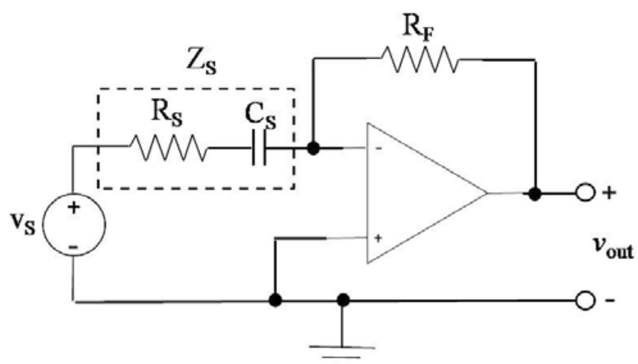

Figure 20: Active High-Pass Filter (AHPF)

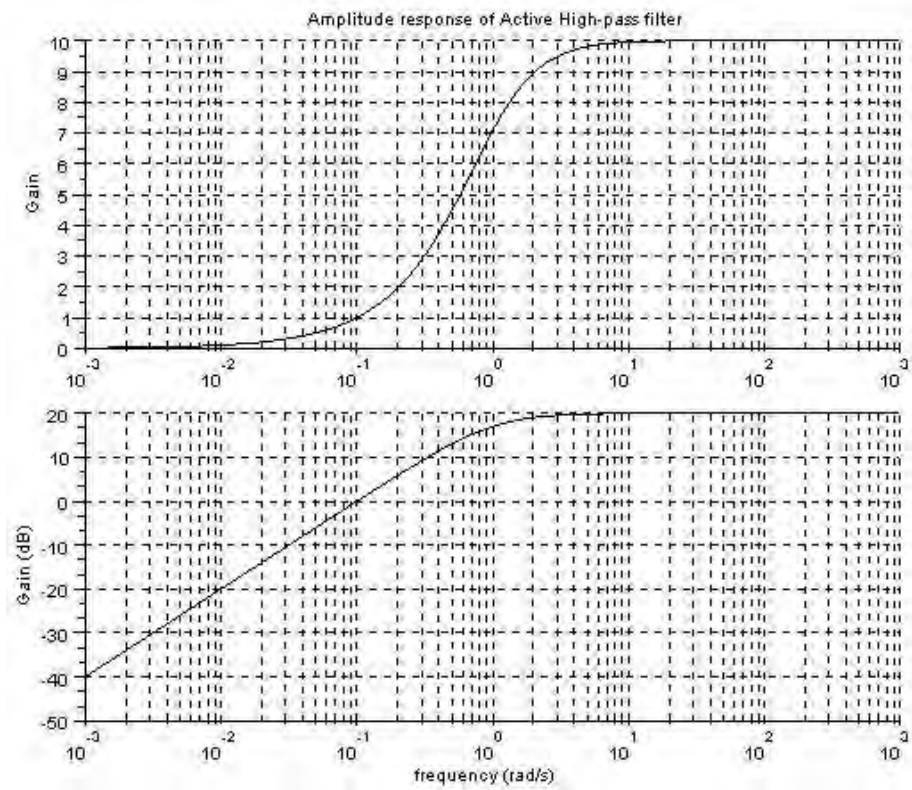

Figure 21: frequency response of AHPF

Active Band-Pass Filter

An *Active Band-Pass Filter (ABPF)* is a circuit that allows a range of frequency to pass through. It can be constructed as shown in Figure 22. The impedances can be computed as

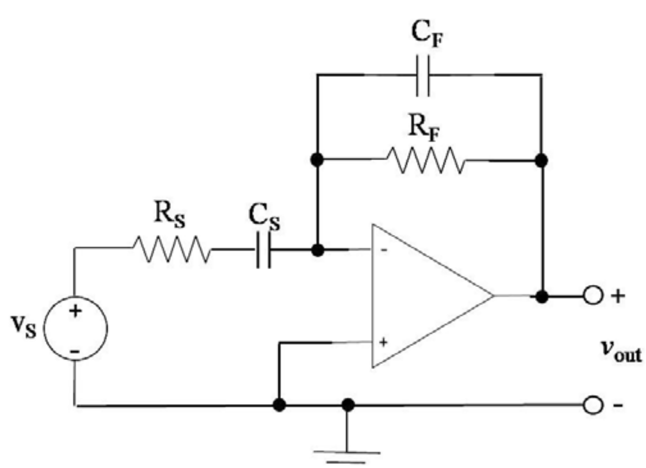

Figure 22: Active Band-Pass Filter (ABPF)

$$Z_F = R_F \parallel \frac{1}{j\omega C_F} = \frac{R_F}{1 + j\omega C_F R_F} \quad (40)$$

$$Z_S = R_S + \frac{1}{j\omega C_S} = \frac{1 + j\omega C_S R_S}{j\omega C_S} \quad (41)$$

Hence the gain of ABPF equals

$$\boxed{A_{BPF}(j\omega) = -\frac{Z_F}{Z_S} = \frac{j\omega C_S R_F}{(1 + j\omega C_F R_F)(1 + j\omega C_S R_S)}} \quad (42)$$

The gain equation of ABPF looks a bit complicated. To analyze and design the filter, we consider 3 important frequencies

$$\omega_1 = \frac{1}{R_F C_S}, \quad \omega_L = \frac{1}{R_F C_F}, \quad \omega_H = \frac{1}{R_S C_S} \quad (43)$$

where ω_1 is called unity gain frequency. ω_L and ω_H are called the low and high cutoff frequency, respectively. Figure 23 shows the frequency response when R_S = 1 KΩ, C_S = 1 μF, C_F = 0.1 μF and R_F = 1 MΩ, which yields ω_1 = 1, ω_L = 10, and ω_H = 1,000 rad/s.

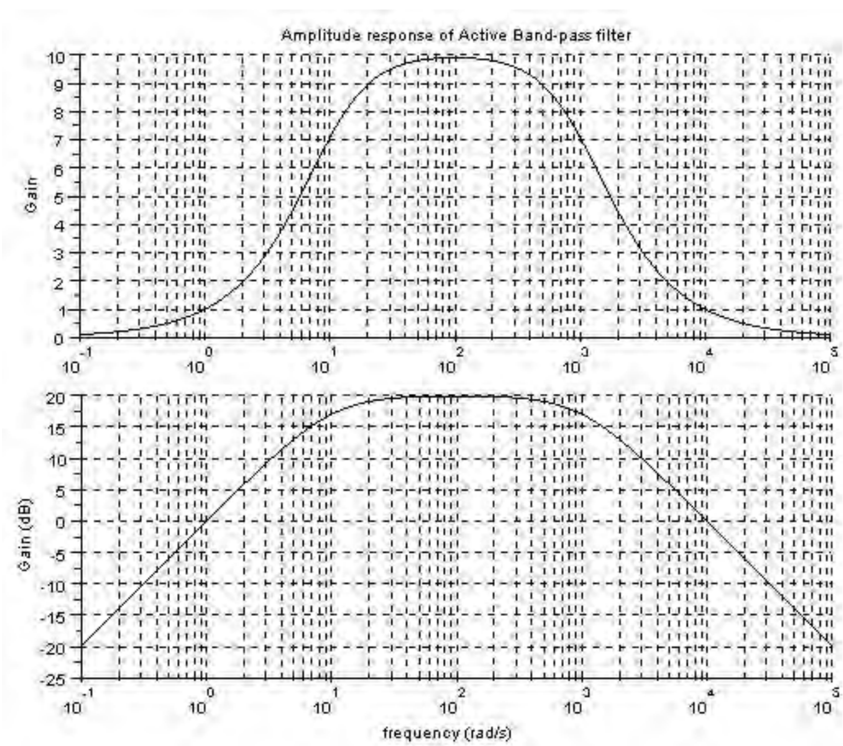

Figure 23: frequency response of ABPF

Summary

In this technical article, we discuss some op-amp basics and circuit examples. The device still plays an important role in the analog domain, due to its availability, low cost, and ease of use. We select only the essence and omit several issues. Users can consult an op-amp datasheet or other references for more comprehensive information.

Though op-amps can be used to construct more complicated circuits, such as a higher-order active filter, or a PID controller. We advise against such approach, since it is more advantageous to implement them digitally. Changing the gain of an op-amp circuit means a physical component has to be replaced. A potentiometer may be deployed in place of a fixed resistor, but one still faces the problem of contact wear and dust. One rule of thumb is, never hard-wire anything you can program.

Reference

G. Rizzoni. Principles and Applications of Electrical Engineering, 5th ed., McGraw-Hill, 2007.